10-3 Circles

Main Ideas

- Write equations of circles.
- Graph circles.

New Vocabulary

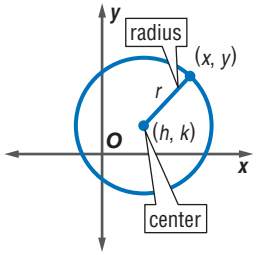
circle
center

GET READY for the Lesson

Radar equipment can be used to detect and locate objects that are too far away to be seen by the human eye. The radar systems at major airports can typically detect and track aircraft up to 45 to 70 miles in any direction from the airport. The boundary of the region that a radar system can monitor can be modeled by a circle.

Equations of Circles A **circle** is the set of all points in a plane that are equidistant from a given point in the plane, called the **center.** Any segment whose endpoints are the center and a point on the circle is a *radius* of the circle.

Assume that (x, y) are the coordinates of a point on the circle at the right. The center is at (h, k), and the radius is r. You can find an equation of the circle by using the Distance Formula.



$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = d \quad \text{Distance Formula}$$

$$\sqrt{(x - h)^2 + (y - k)^2} = r \quad (x_1, y_1) = (h, k), \ (x_2, y_2) = (x, y), \ d = r$$

$$(x - h)^2 + (y - k)^2 = r^2 \quad \text{Square each side.}$$

This is the standard form of the equation of a circle.

KEY CONCEPT *Equation of a Circle*

The equation of a circle with center (h, k) and radius r units is $(x - h)^2 + (y - k)^2 = r^2$.

You can use the standard form of the equation of a circle to write an equation for a circle given its center and the radius or diameter. Recall that a segment that passes through the center of a circle whose endpoints are on the circle is a diameter.

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Real-World EXAMPLE Write an Equation Given the Radius

Real-World Link

WiFi technology uses radio waves to transmit data. It allows high-speed access to the Internet without the use of cables.

Source: wifiphone.org

1 **DELIVERY** **An appliance store offers free delivery within 35 miles of the store. The Jackson store is located 100 miles north and 45 miles east of the corporate office. Write an equation to represent the delivery boundary of the Jackson store if the origin of the coordinate system is the corporate office.**

Words Since the corporate office is at (0, 0), the Jackson store is at (45, 100). The boundary of the delivery region is the circle centered at (45, 100) with radius 35 miles.

Variables

$$(x - h)^2 + (y - k)^2 = r^2$$ Equation of a circle

Equation

$$[x - (-45)]^2 + (y - 100)^2 = 35^2$$ $(h, k) = (45, 100), r = 35$

$$(x - 45)^2 + (y - 100)^2 = 1225$$ Simplify.

CHECK Your Progress

1. WIFI A certain wireless transmitter has a range of thirty miles in any direction. If a WiFi phone is 4 miles south and 3 miles west of the headquarters building, write an equation to represent the area that the phone can communicate via the WiFi system.

EXAMPLE Write an Equation Given a Diameter

2 **Write an equation for a circle if the endpoints of a diameter are at (5, 4) and (−2, −6).**

$$(h, k) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$ Midpoint Formula

$$= \left(\frac{5 + (-2)}{2}, \frac{4 + (-6)}{2}\right)$$ $(x_1, y_1) = (5, 4), (x_2, y_2) = (-2, -6)$

$$= \left(\frac{3}{2}, \frac{-2}{2}\right)$$ Add.

$$= \left(\frac{3}{2}, -1\right)$$ Simplify.

Now find the radius.

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$ Distance Formula

$$= \sqrt{\left(\frac{3}{2} - 5\right)^2 + (-1 - 4)^2}$$ $(x_1, y_1) = (5, 4), (x_2, y_2) = \left(\frac{3}{2}, -1\right)$

$$= \sqrt{\left(-\frac{7}{2}\right)^2 + (-5)^2}$$ Subtract.

$$= \sqrt{\frac{149}{4}}$$ Simplify.

The radius of the circle is $\sqrt{\frac{149}{4}}$ units, so $r^2 = \frac{149}{4}$.

Substitute h, k, and r^2 into the standard form of the equation of a circle.

An equation of the circle is $\left(x - \frac{3}{2}\right)^2 + (y + 1)^2 = \frac{149}{4}$.

CHECK Your Progress

2. Write an equation for a circle if the endpoints of a diameter are at (3, −3) and (1, 5).

Graph Circles You can use symmetry to help you graph circles.

EXAMPLE Graph an Equation in Standard Form

3 **Find the center and radius of the circle with equation $x^2 + y^2 = 25$. Then graph the circle.**

The center of the circle is at (0, 0), and the radius is 5.

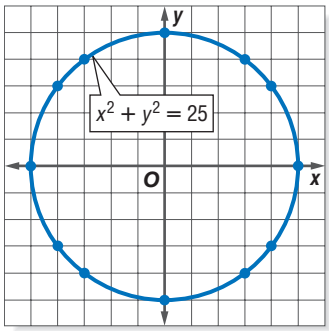
The table lists some integer values for x and y that satisfy the equation.

x	y
0	5
3	4
4	3
5	0

Since the circle is centered at the origin, it is symmetric about the y-axis. Therefore, the points at (−3, 4), (−4, 3), and (−5, 0) lie on the graph.

The circle is also symmetric about the x-axis, so the points at (−4, −3), (−3, −4), (0, −5), (3, −4), and (4, −3) lie on the graph.

Graph all of these points and draw the circle that passes through them.



Cross-Curricular Project

The epicenter of an earthquake can be located by using the equation of a circle. Visit algebra2.com to continue work on your project.

CHECK Your Progress

3. Find the center and radius of the circle with equation $x^2 + y^2 = 81$. Then graph the circle.

Circles with centers that are not at (0, 0) can be graphed using translations. The equation $(x - h)^2 + (y - k)^2 = r^2$ is obtained from the equation $x^2 + y^2 = r^2$ by replacing x with $x - h$ and y with $y - k$. So, the graph of $(x - h)^2 + (y - k)^2 = r^2$ is the graph of $x^2 + y^2 = r^2$ translated h units to the right or left and k units up or down.

EXAMPLE Graph an Equation Not in Standard Form

4 **Find the center and radius of the circle with equation $x^2 + y^2 - 4x + 8y - 5 = 0$. Then graph the circle.**

Complete the squares.

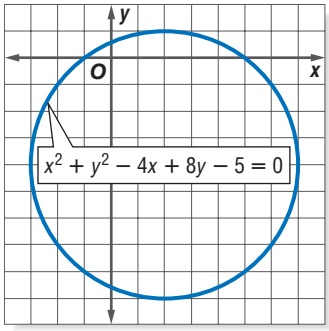
$$x^2 + y^2 - 4x + 8y - 5 = 0$$

$$x^2 - 4x + \blacksquare + y^2 + 8y + \blacksquare = 5 + \blacksquare + \blacksquare$$

$$x^2 - 4x + 4 + y^2 + 8y + 16 = 5 + 4 + 16$$

$$(x - 2)^2 + (y + 4)^2 = 25$$

The center of the circle is at (2, −4), and the radius is 5. In the equation from Example 3, x has been replaced by $x - 2$, and y has been replaced by $y + 4$. The graph is the graph from Example 3 translated 2 units to the right and down 4 units.



CHECK Your Progress

4. Find the center and radius of the circle with equation $x^2 + y^2 + 4x - 10y - 7 = 0$. Then graph the circle.

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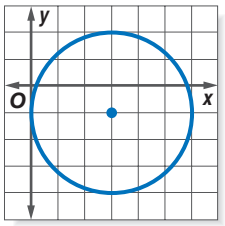
CHECK Your Understanding

Example 1 (p. 575)

1. Write an equation for the graph at the right.



AEROSPACE **For Exercises 2 and 3, use the following information.**
In order for a satellite to remain in a circular orbit above the same spot on Earth, the satellite must be 35,800 kilometers above the equator.

2. Write an equation for the orbit of the satellite. Use the center of Earth as the origin and 6400 kilometers for the radius of Earth.

3. Draw a labeled sketch of Earth and the orbit to scale.

Example 2 (p. 575)

Write an equation for the circle that satisfies each set of conditions.

4. center $(-1, -5)$, radius 2 units

5. endpoints of a diameter at $(-4, 1)$ and $(4, -5)$

6. endpoints of a diameter at $(2, -2)$ and $(-2, -6)$

Examples 3, 4 (p. 576)

Find the center and radius of the circle with the given equation. Then graph the circle.

7. $(x - 4)^2 + (y - 1)^2 = 9$ **8.** $x^2 + (y - 14)^2 = 34$ **9.** $(x - 4)^2 + y^2 = \frac{16}{25}$

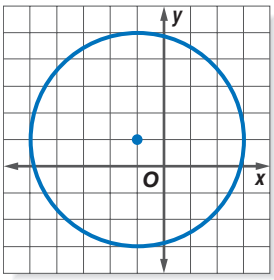
10. $\left(x + \frac{2}{3}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{8}{9}$ **11.** $x^2 + y^2 + 8x - 6y = 0$ **12.** $x^2 + y^2 + 4x - 8 = 0$

Exercises

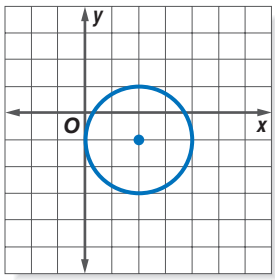
HOMEWORK HELP	
For Exercises	See Examples
13–17	1
18, 19	2
20–25	3
26–31	4

Write an equation for each graph.

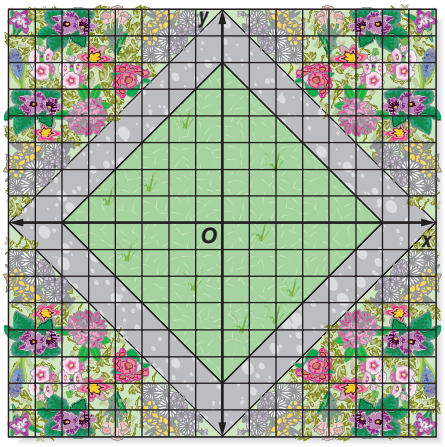
13.



14.



15. LANDSCAPING The design of a garden is shown at the right. A pond is to be built in the center region. What is the equation of the largest circular pond centered at the origin that would fit within the walkways?



Write an equation for the circle that satisfies each set of conditions.

16. center (0, 3), radius 7 units

17. center (−8, 7), radius $\frac{1}{2}$ unit

18. endpoints of a diameter at (−5, 2) and (3, 6)

19. endpoints of a diameter at (11, 18) and (−13, −19)

Find the center and radius of the circle with the given equation. Then graph the circle.

20. $x^2 + (y + 2)^2 = 4$

21. $x^2 + y^2 = 144$

22. $(x - 3)^2 + (y - 1)^2 = 25$

23. $(x + 3)^2 + (y + 7)^2 = 81$

24. $(x - 3)^2 + y^2 = 16$

25. $(x - 3)^2 + (y + 7)^2 = 50$

26. $x^2 + y^2 + 6y = -50 - 14x$

27. $x^2 + y^2 - 6y - 16 = 0$

28. $x^2 + y^2 + 2x - 10 = 0$

29. $x^2 + y^2 - 18x - 18y + 53 = 0$

30. $x^2 + y^2 + 9x - 8y + 4 = 0$

31. $x^2 + y^2 - 3x + 8y = 20$

Write an equation for the circle that satisfies each set of conditions.

32. center (8, −9), passes through (21, 22)

33. center $\left(-\sqrt{13}, 42\right)$, passes through the origin

34. center at (−8, −7), tangent to y-axis

35. center at (4, 2), tangent to x-axis

36. center in the first quadrant; tangent to $x = -3$, $x = 5$, and the x-axis

37. center in the second quadrant; tangent to $y = -1$, $y = 9$, and the y-axis

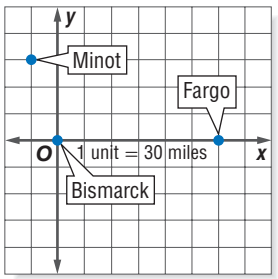
Real-World Link

Southern California has about 10,000 earthquakes per year. Most are too small to be felt.

Source: earthquake.usgs.gov

38. EARTHQUAKES The Rose Bowl is located about 35 miles west and about 40 miles north of downtown Los Angeles. Suppose an earthquake occurs with its epicenter about 55 miles from the stadium. Assume that the origin of a coordinate plane is located at the center of downtown Los Angeles. Write an equation for the set of points that could be the epicenter of the earthquake.

39. RADIO The diagram at the right shows the relative locations of some cities in North Dakota. The x-axis represents Interstate 94. While driving west on the highway, Doralina is listening to a radio station broadcasting from Minot. She estimates the range of the signal to be 120 miles. How far west of Bismarck will she be able to pick up the signal?



Graphing Calculator

For Exercises 40–43, use the following information.

Since a circle is not the graph of a function, you cannot enter its equation directly into a graphing calculator. Instead, you must solve the equation for y. The result will contain a $\pm$ symbol, so you will have two functions.

40. Solve $(x + 3)^2 + y^2 = 16$ for y.

41. What two functions should you enter to graph the given equation?

42. Graph $(x + 3)^2 + y^2 = 16$ on a graphing calculator.

43. Solve $(x + 3)^2 + y^2 = 16$ for x. What parts of the circle do the two expressions for x represent?

EXTRA PRACTICE
See pages 912, 935.

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H.O.T. Problems

44. OPEN ENDED Write an equation for a circle with center at (6, −2).

45. REASONING Write $x^2 + y^2 + 6x - 2y - 54 = 0$ in standard form by completing the square. Describe the transformation that can be applied to the graph of $x^2 + y^2 = 64$ to obtain the graph of the given equation.

46. FIND THE ERROR Juwan says that the circle with equation $(x - 4)^2 + y^2 = 36$ has radius 36 units. Lucy says that the radius is 6 units. Who is correct? Explain your reasoning.

47. CHALLENGE A circle has its center on the line with equation $y = 2x$. It passes through (1, −3) and has a radius of $\sqrt{5}$ units. Write an equation of the circle.

48. ***Writing in Math*** Use the information about radar equipment on page 574 to explain why circles are important in air traffic control. Include an equation of the circle that determines the boundary of the region where planes can be detected if the range of the radar is 50 miles and the radar is at the origin.

STANDARDIZED TEST PRACTICE

49. ACT/SAT What is the center of the circle with equation $x^2 + y^2 - 10x + 6y + 27 = 0$?

A (−10, 6)

B (1, 1)

C (10, −6)

D (5, −3)

50. REVIEW If the surface area of a cube is increased by a factor of 9, how is the length of the side of the cube changed?

F It is 2 times the original length.

G It is 3 times the original length.

H It is 4 times the original length.

J It is 5 times the original length.

Spiral Review

Identify the coordinates of the vertex and focus, the equations of the axis of symmetry and directrix, and the direction of opening of the parabola with the given equation. Then find the length of the latus rectum and graph the parabola. (Lesson 10-2)

51. $x = -3y^2 + 1$ **52.** $y + 2 = -(x - 3)^2$ **53.** $y = x^2 + 4x$

Find the midpoint of the line segment with endpoints having the given coordinates. (Lesson 10-1)

54. (5, −7), (3, −1) **55.** (2, −9), (−4, 5) **56.** (8, 0), (−5, 12)

Find all of the rational zeros for each function. (Lesson 6-8)

57. $f(x) = x^3 + 5x^2 + 2x - 8$ **58.** $g(x) = 2x^3 - 9x^2 + 7x + 6$

59. PHOTOGRAPHY The perimeter of a rectangular picture is 86 inches. Twice the width exceeds the length by 2 inches. What are the dimensions of the picture? (Lesson 3-2)

GET READY for the Next Lesson

PREREQUISITE SKILL Solve each equation. Assume that all variables are positive. (Lesson 5-5)

60. $c^2 = 13^2 - 5^2$ **61.** $c^2 = 10^2 - 8^2$ **62.** $(\sqrt{7})^2 = a^2 - 3^2$ **63.** $4^2 = 6^2 - b^2$